

**Title:** Negative and Rational Exponents  
**Class:** Math 107 or Math 111 or Math 120 or Math 137  
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**Instructions to Tutor:** Read instructions and follow all steps for each problem exactly as given.  
**Keywords/Tags:** negative exponents, rational exponents, simplifying exponents

## Negative and Rational Exponents

**Purpose:** This is intended to refresh your skills in rewriting or simplifying expressions with negative exponents and with rational exponents. It is also intended to help you clarify and distinguish between these two types of exponents.

**Activity:** Work through the following activity and examples. Do all of the practice problems before consulting with a tutor.

Exponents tell you what to do with the base in a product (the answer from multiplying).

$2^3 = 2 \times 2 \times 2 = 8$       The exponent 3 tells you that you need 3 factors of the base 2 to get the product of 8.

$2^4 = 2 \times 2 \times 2 \times 2 = 16$        $\div 2$

$2^3 = 2 \times 2 \times 2 = 8$        $\div 2$

$2^2 = 2 \times 2 = 4$        $\div 2$

$2^1 = 2 = 2$        $\div 2$

$2^0 = 1 = 1$        $\div 2$

$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$        $\div 2$

$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$        $\div 2$

$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$        $\div 2$

Recall that  $\div 2$  is the same as multiplying by the reciprocal of 2, which is  $\frac{1}{2}$ .

↑ Notice the pattern of the exponents.

- A **negative exponent** tells you that you are to either
  - divide by the base and rewrite the exponent as positive
  - OR • multiply by the reciprocal

$$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

$$7^{-1} = \frac{1}{7^1} = \frac{1}{7}$$

$$9^{-2} = \frac{1}{9^2} = \frac{1}{81}$$

**Rule:** for  $b > 0$ ,  $b^{-n} = \frac{1}{b^n}$

**Practice:**

$3^{-3} =$

$4^{-1} =$

$6^{-2} =$

$10^{-4} =$

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Next, consider  $\frac{1}{2^{-3}} = \frac{1}{\frac{1}{2^3}} = 1 \div \frac{1}{2^3} = 1 \cdot \frac{2^3}{1} = \frac{2^3}{1} = 2^3 = 8$

$\frac{1}{2^{-4}} = \frac{2^4}{1} = 2^4 = 16$

$\frac{1}{5^{-2}} = \frac{5^2}{1} = 5^2 = 25$

$\frac{1}{7^{-1}} = \frac{7^1}{1} = 7^1 = 7$

$\frac{1}{9^{-2}} = \frac{9^2}{1} = 9^2 = 81$

**Rule:** for  $b > 0$ ,  $\frac{1}{b^{-n}} = \frac{b^n}{1} = b^n$

**Practice:**

$\frac{1}{3^{-3}} =$

$\frac{1}{4^{-1}} =$

$\frac{1}{6^{-2}} =$

$\frac{1}{10^{-4}} =$

Negative exponents mean “take the reciprocal of the base to the positive exponent.”

$2^{-3} = \left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \frac{1}{2^3} = \frac{1}{8}$

$\left(\frac{1}{2}\right)^{-3} = \left(\frac{2}{1}\right)^3 = \frac{2^3}{1^3} = \frac{2^3}{1} = 2^3 = 8$

$\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2 = \frac{5^2}{3^2} = \frac{25}{9}$

**Rule:** for  $a > 0$ ,  $b > 0$ ,  $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

**Practice:**

$\left(\frac{2}{7}\right)^{-2} =$

$\left(\frac{4}{3}\right)^{-3} =$

$\left(\frac{1}{10}\right)^{-4} =$

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- **Rational exponents** (exponents which are fractions) are telling you to undo a power:

$$25^{\frac{1}{2}} = \sqrt[2]{25} = \sqrt{25} = 5, \text{ since } 5^{\frac{2}{1}} = 5^2 = 25$$

$$8^{\frac{1}{3}} = \sqrt[3]{8} = 2, \text{ since } 2^{\frac{3}{1}} = 2^3 = 8$$

$$16^{\frac{1}{4}} = \sqrt[4]{16} = 2, \text{ since } 2^{\frac{4}{1}} = 2^4 = 16$$

$$\left(\frac{1}{16}\right)^{\frac{1}{2}} = \sqrt{\left(\frac{1}{16}\right)} = \frac{1}{4}, \text{ since } \left(\frac{1}{4}\right)^{\frac{2}{1}} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

**Rule:** for  $b > 0$ ,  $b^{\frac{1}{n}} = \sqrt[n]{b} = a$  where  $a^n = b$ .

$b^{\frac{1}{n}}$  says to find a number  $a$  that satisfies  $a^n = b$ . You are trying to undo  $a^n = b$ .

**Practice:**

$$9^{\frac{1}{2}} =$$

$$125^{\frac{1}{3}} =$$

$$81^{\frac{1}{4}} =$$

$$\left(\frac{1}{100}\right)^{\frac{1}{2}} =$$

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- What about **negative rational exponents**? The exponent is always telling you what to do to the base. A negative rational exponent is telling you to do two separate things to the base:

- take the reciprocal
- AND • undo the power.

$$9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{3}$$

$$25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}} = \frac{1}{5}$$

$$\frac{1}{8^{-\frac{1}{3}}} = \frac{8^{\frac{1}{3}}}{1} = 2$$

$$\left(\frac{1}{10000}\right)^{-\frac{1}{4}} = \left(\frac{10000}{1}\right)^{\frac{1}{4}} = \frac{10}{1} = 10$$

A common mistake is to think that a negative exponent means to take the reciprocal of the exponent, not the base. For example,  $8^{-3} \neq 8^{\frac{1}{3}}$ , since  $8^{-3} = \frac{1}{8^3} = \frac{1}{512}$  but  $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$ .

**Practice:**

$$4^{-\frac{1}{2}} =$$

$$81^{-\frac{1}{2}} =$$

$$81^{-\frac{1}{4}} =$$

$$\left(\frac{1}{9}\right)^{-\frac{1}{2}} =$$

**More Practice:**

$4^{-2} =$

$4^{\frac{1}{2}} =$

$4^{-\frac{1}{2}} =$

$16^{-4} =$

$16^{\frac{1}{4}} =$

$16^{-\frac{1}{4}} =$

$\left(\frac{8}{27}\right)^{-3} =$

$\left(\frac{8}{27}\right)^{\frac{1}{3}} =$

$\left(\frac{8}{27}\right)^{-\frac{1}{3}} =$

$\left(\frac{8x^3}{27y^9}\right)^{-3} =$

$\left(\frac{8x^3}{27y^9}\right)^{\frac{1}{3}} =$

$\left(\frac{8x^3}{27y^9}\right)^{-\frac{1}{3}} =$

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**Review:** Meet with a tutor to verify your work on this worksheet and discuss some of the areas that were more challenging for you. If necessary, choose more problems from the homework to practice and discuss with the tutor.

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**For Tutor Use:** Please check the appropriate statement:

\_\_\_\_\_ Student has completed worksheet but may need further assistance. Recommend a follow-up with the instructor.

\_\_\_\_\_ Student has mastered topic.